

Emergent Cosmological Constant from Infrared Entanglement Equilibrium

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Abstract

We present a self-contained infrared resolution of the cosmological constant problem. The cosmological constant is shown to be an emergent, IR-determined quantity fixed by an entanglement-based consistency condition on a finite causal diamond. Vacuum energy cancels identically in the modular energy, eliminating radiative instability without modifying Einstein's equations. The resulting cosmological constant is naturally of order H_0^2 without fine-tuning.

1 Introduction

The cosmological constant problem has long been regarded as one of the most serious conceptual challenges at the interface of gravity and quantum field theory. Observationally, cosmic acceleration is well described by a small, positive cosmological constant Λ , corresponding to an energy scale of order H_0^2 in units where $c = 1$. From the perspective of quantum field theory, however, vacuum fluctuations are expected to generate contributions to the energy density many orders of magnitude larger, leading to an apparent fine-tuning problem of order 10^{120} .

This tension rests on two distinct assumptions. The first is that Λ is a fundamental parameter of the gravitational action, sensitive to ultraviolet physics and subject to radiative corrections. The second is that constant vacuum energy necessarily gravitates and contributes directly to the effective cosmological constant. Together, these assumptions imply an extreme mismatch between ultraviolet and infrared scales.

In this work we adopt a different perspective. We argue that the cosmological constant is not a microscopic parameter to be renormalized, but an emergent infrared quantity determined by the entanglement structure of quantum fields on cosmological scales. The central object in our analysis is the causal diamond of a late-time comoving observer, which provides a natural, operationally defined infrared regulator in cosmology. Within this finite region, physically relevant gravitational information is encoded not in the absolute value of the vacuum energy, but in vacuum-subtracted quantities defined relative to a reference state.

Our approach is based on the relative entropy between the physical state of quantum fields and a reference vacuum restricted to the causal diamond. Relative entropy furnishes a well-defined, positive, and ultraviolet-insensitive measure of distinguishability between quantum states. Crucially, the associated modular energy is defined relative to the vacuum, so that constant shifts of the stress tensor cancel identically. In this modular sense, vacuum energy does not gravitate, and radiative instability of Λ is eliminated without modifying Einstein's equations or introducing additional fields.

The remaining question is then infrared in nature: how is the value of Λ selected once ultraviolet sensitivity has been removed? We propose that Λ is fixed by an entanglement-based consistency

condition on the causal diamond, equating the vacuum-subtracted modular energy of excitations to the generalized entropy associated with the diamond boundary. This condition selects a unique horizon scale R_* , which in turn determines an effective cosmological constant $\Lambda_{\text{eff}} \sim 1/R_*^2$.

The central claim of this paper is modest but precise. We do not attempt to derive a microscopic value of Λ from Planck-scale physics. Rather, we show that: (i) Λ is determined by infrared, semi-classical observables defined on a finite causal diamond; (ii) constant vacuum-energy contributions decouple identically from this determination; and (iii) in matter+ Λ cosmologies the resulting Λ_{eff} is naturally of order H_0^2 , up to a coefficient of order unity, without fine-tuning.

In this sense, the traditional cosmological constant problem is resolved. The ultraviolet naturalness crisis arises from an incorrect identification of the source of Λ . Once Λ is recognized as an infrared, entanglement-controlled quantity rather than a renormalized vacuum energy, the apparent 10^{120} discrepancy disappears. What remains is a well-posed infrared selection problem, governed by the geometry and entanglement structure of cosmological causal diamonds.

The paper is organized as follows. In Section 2 we introduce causal diamonds in FLRW spacetimes and identify the relevant infrared scale. Section 3 reviews relative entropy and modular energy, emphasizing the exact cancellation of vacuum-energy contributions. In Section 4 we formulate the infrared entanglement equilibrium condition that fixes the horizon scale. Section 5 shows explicitly that the infrared excitation density obtained by causal-diamond averaging in Λ CDM is of order the present critical density. We conclude in Section 6 with a discussion of the scope, limitations, and implications of the framework. Technical details are collected in the appendices.

2 Causal Diamonds and the Infrared Scale

In cosmology, physically meaningful infrared observables are necessarily observer-dependent. A natural and operationally well-defined infrared regulator is provided by the *causal diamond* associated with a given worldline. The causal diamond captures precisely the spacetime region that can both influence and be influenced by a given observer over a finite interval of proper time.

We consider a comoving observer in a spatially flat FLRW spacetime with metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2. \quad (1)$$

Given an initial time t_i and a final time t_f , the causal diamond $\mathcal{D}(t_f)$ is defined as

$$\mathcal{D}(t_f) = J^+(p_{t_i}) \cap J^-(p_{t_f}), \quad (2)$$

where J^\pm denote the causal future and past, respectively, and p_t denotes the observer's worldline.

At an intermediate time t , the spatial section of the diamond is a ball of physical radius

$$R(t; t_f) = a(t) \min(\eta(t) - \eta(t_i), \eta(t_f) - \eta(t)), \quad (3)$$

where $\eta(t)$ is conformal time. The radius grows from zero at $t = t_i$, reaches a maximum at an intermediate time, and then shrinks back to zero at $t = t_f$.

In matter+ Λ cosmologies, taking the late-time limit $t_f \rightarrow \infty$ yields a causal diamond with a well-defined maximal spatial slice. The corresponding maximal radius R_{max} is finite and is of order the de Sitter horizon scale. This maximal radius defines the relevant infrared length scale for the observer. Throughout this work we identify

$$R_* \equiv R_{\text{max}} \quad (4)$$

as the infrared scale entering the entanglement equilibrium condition.

The use of a finite causal diamond is essential. First, it avoids the infrared ambiguities associated with infinite spatial volumes in FLRW spacetimes. Second, it provides an operational notion of coarse-graining tied to a physical observer. Finally, in the presence of a late-time event horizon, the causal diamond naturally saturates to a finite size, allowing infrared quantities to approach fixed-point values.

In the following sections we will evaluate physical quantities—entropy, modular energy, and averaged excitation density—on the causal diamond and, in particular, on its maximal spatial slice. Detailed expressions for the diamond geometry and the associated spacetime integrals are collected in Appendix A. For the purposes of the main text, it suffices to note that the causal diamond furnishes a canonical infrared scale R_* that depends only on the large-scale cosmological evolution and not on ultraviolet physics.

3 Relative Entropy and Modular Energy

The central quantity underlying our analysis is the relative entropy between two quantum states restricted to the algebra of observables associated with a finite causal diamond. Relative entropy provides a natural, state-dependent measure of distinguishability that is ultraviolet finite and strictly positive.

Given a physical state ρ and a reference state σ , both restricted to the causal diamond \mathcal{D} , the relative entropy is defined as

$$S_{\text{rel}}(\rho||\sigma) = \text{Tr}(\rho \log \rho) - \text{Tr}(\rho \log \sigma). \quad (5)$$

It admits the equivalent expression

$$S_{\text{rel}} = \Delta\langle K_\sigma \rangle - \Delta S, \quad (6)$$

where

$$K_\sigma \equiv -\log \sigma \quad (7)$$

is the modular Hamiltonian associated with the reference state, ΔS is the difference in von Neumann entropy, and $\Delta\langle K_\sigma \rangle$ is the expectation value of the modular Hamiltonian in the physical state relative to the reference state.

A crucial property of the modular Hamiltonian is that it is defined *relative to the reference state*. As a result, any constant shift of the stress-energy tensor,

$$T_{\mu\nu} \longrightarrow T_{\mu\nu} + \rho_{\text{vac}} g_{\mu\nu}, \quad (8)$$

cancels identically in $\Delta\langle K_\sigma \rangle$. This cancellation holds independently of the magnitude of ρ_{vac} , the field content, or the ultraviolet cutoff. In this precise modular sense, constant vacuum energy does not gravitate.

This observation eliminates the radiative instability that underlies the traditional cosmological constant problem. Vacuum energy may contribute to the expectation value of the stress tensor in semiclassical Einstein equations, but it does not enter the vacuum-subtracted modular energy that governs the infrared entanglement balance on the causal diamond. The determination of the cosmological constant in our framework is therefore insensitive to ultraviolet physics.

For spherical regions in flat spacetime, the modular Hamiltonian of the vacuum is known exactly and takes a local form. For a ball of radius R ,

$$\Delta\langle K_\sigma \rangle = 2\pi \int_{|\vec{x}|\leq R} d^3x \frac{R^2 - |\vec{x}|^2}{2R} \Delta T_{00}(\vec{x}). \quad (9)$$

For homogeneous excitations with energy density ρ , this expression scales as $\Delta\langle K_\sigma \rangle \propto \rho R^4$.

In an expanding FLRW spacetime, the exact modular Hamiltonian for a causal diamond is not known in closed form. However, the leading infrared contribution is dominated by the maximal spatial slice of the diamond, where curvature effects are parametrically small. For the purposes of determining the infrared scale R_* and the associated cosmological constant, it suffices to retain the leading R^4 scaling of the modular energy with an effective excitation density averaged over the diamond. Corrections due to curvature and non-conformal fields affect only order-unity coefficients and do not alter the infrared structure of the equilibrium condition. A more detailed discussion of this approximation is given in Appendix B.

In summary, relative entropy provides a natural framework in which the relevant gravitational energy is automatically vacuum-subtracted and ultraviolet insensitive. The only quantities entering the infrared determination of the cosmological constant are the modular energy of excitations above the vacuum and the generalized entropy associated with the causal diamond boundary.

4 Infrared Entanglement Equilibrium

Having identified the causal diamond as the relevant infrared region and established the vacuum-subtracted nature of modular energy, we now formulate the principle that fixes the infrared scale R_* and thereby determines the effective cosmological constant.

The key object is the relative entropy on the causal diamond,

$$S_{\text{rel}}(R) = \Delta\langle K_\sigma(R) \rangle - \Delta S(R), \quad (10)$$

viewed as a function of the characteristic size R of the diamond. Here $\Delta\langle K_\sigma(R) \rangle$ denotes the modular energy of excitations above the reference vacuum, while $\Delta S(R)$ is the associated generalized entropy, dominated at leading order by the area term of the diamond boundary.

Proposition: Infrared entanglement equilibrium

Proposition. In a matter+ Λ FLRW spacetime with a late-time event horizon, the relative entropy $S_{\text{rel}}(R)$ defined on the causal diamond possesses a unique stationary point at a finite radius $R = R_*$, corresponding to an infrared entanglement equilibrium. This stationary point fixes the effective cosmological constant via

$$\Lambda_{\text{eff}} = \frac{3}{R_*^2}. \quad (11)$$

Proof sketch

The proof relies on the scaling behavior of the two terms entering $S_{\text{rel}}(R)$.

At small R , the modular energy scales as $\Delta\langle K_\sigma(R) \rangle \sim R^4$ for homogeneous excitations, while the entropy term scales as $\Delta S(R) \sim R^2$. Consequently, $S_{\text{rel}}(R) > 0$ and increases with R for sufficiently small diamonds.

At large R , the presence of a late-time event horizon ensures that the causal diamond saturates to a finite maximal size. The excitation energy density averaged over the diamond dilutes due to cosmic expansion, so that the modular energy grows no faster than R^2 . By contrast, the generalized entropy associated with the boundary grows strictly as R^2 . As a result, $S_{\text{rel}}(R)$ decreases for sufficiently large R .

By continuity, $S_{\text{rel}}(R)$ must therefore admit at least one stationary point at finite R . Uniqueness follows from the fact that the entropy term is strictly convex in R , while the modular energy is at

most marginally growing in the infrared. The stationary point is a minimum of $S_{\text{rel}}(R)$, equivalently a maximum of the functional $I(R) \equiv -S_{\text{rel}}(R)$.

Equilibrium condition

At the stationary point $R = R_*$, the equilibrium condition reads

$$\left. \frac{d}{dR} \Delta \langle K_\sigma(R) \rangle \right|_{R_*} = \left. \frac{d}{dR} \Delta S(R) \right|_{R_*}. \quad (12)$$

Using the leading-order expressions for homogeneous excitations,

$$\Delta \langle K_\sigma(R) \rangle = \frac{8\pi^2}{15} \rho_{\text{IR}} R^4, \quad \Delta S(R) = \frac{\pi}{G} R^2, \quad (13)$$

this condition yields

$$R_*^2 = \frac{15}{16\pi} \frac{1}{G \rho_{\text{IR}}}. \quad (14)$$

The corresponding effective cosmological constant is therefore

$$\Lambda_{\text{eff}} = \frac{3}{R_*^2} = \frac{16\pi}{5} G \rho_{\text{IR}}, \quad (15)$$

up to an order-unity numerical coefficient that depends on the precise definition of generalized entropy on the diamond boundary.

Interpretation

The infrared entanglement equilibrium condition does not modify Einstein's equations and does not introduce new dynamical fields. Rather, it supplements semiclassical gravity with a global infrared consistency requirement that fixes the horizon scale in terms of vacuum-subtracted excitation energy. The resulting cosmological constant is determined entirely by infrared physics and is insensitive to ultraviolet vacuum contributions.

The remaining task is to determine the infrared excitation density ρ_{IR} in realistic cosmologies. In the next section we show that in Λ CDM this quantity is naturally of order the present critical density, with no large or small hierarchies.

5 Infrared Excitation Density in Λ CDM

The infrared entanglement equilibrium condition derived in the previous section fixes the cosmological constant in terms of a single quantity: the excitation energy density ρ_{IR} averaged over the causal diamond. We now show that in a realistic Λ CDM cosmology this quantity is automatically of order the present critical density, with no large or small hierarchies.

We define the infrared excitation density as the spacetime average of the homogeneous matter density over the causal diamond \mathcal{D} ,

$$\rho_{\text{IR}} \equiv \frac{1}{V_4} \int_{\mathcal{D}} d^4x \sqrt{-g} \rho_m(t), \quad V_4 = \int_{\mathcal{D}} d^4x \sqrt{-g}. \quad (16)$$

This definition is fully covariant and involves no reference to ultraviolet cutoffs or vacuum contributions. Only excitations above the vacuum enter.

In a spatially flat FLRW spacetime, the integral can be written as a weighted average over the scale factor,

$$\rho_{\text{IR}} = \frac{\int da W(a) \rho_m(a)}{\int da W(a)}, \quad W(a) = \frac{a^2}{H(a)} R^3(a), \quad (17)$$

where $R(a)$ is the physical radius of the causal diamond at scale factor a . The weight $W(a)$ is purely geometric and encodes the spacetime volume of each spatial slice of the diamond.

A key observation is that the function $W(a)$ is sharply peaked near the epoch at which the causal diamond reaches its maximal spatial extent. In matter+ Λ cosmologies this occurs close to the matter– Λ transition. As a result, the weighted average is dominated by a narrow range of scale factors around a characteristic value a_* of order unity.

Evaluating the matter density at this epoch,

$$\rho_m(a_*) = \rho_{m0} a_*^{-3}, \quad (18)$$

one finds that $\rho_m(a_*)$ is generically of order the present critical density $\rho_{\text{crit},0}$. Corrections arising from the finite width of the peak in $W(a)$ modify this estimate only by numerical factors of order unity. A detailed derivation of this result is given in Appendix A.

We therefore arrive at the central conclusion of this section:

$$\rho_{\text{IR}} = c \rho_{\text{crit},0}, \quad c = \mathcal{O}(1), \quad (19)$$

independently of ultraviolet physics, vacuum energy contributions, or the precise values of the cosmological parameters, provided the universe admits a late-time de Sitter phase.

Substituting this result into the equilibrium condition derived in the previous section yields

$$\Lambda_{\text{eff}} = \frac{16\pi}{5} G \rho_{\text{IR}} \sim H_0^2, \quad (20)$$

up to an order-unity coefficient. The observed smallness of the cosmological constant thus reflects the infrared scale set by the present Hubble expansion, rather than a fine-tuned cancellation of vacuum energies.

This completes the determination of Λ within the infrared entanglement equilibrium framework. No additional dynamical assumptions or ultraviolet input are required.

6 Discussion

The framework developed in this work resolves the cosmological constant problem by reinterpreting the role of Λ within semiclassical gravity. Rather than a fundamental parameter of the gravitational action or a renormalized vacuum energy, the cosmological constant emerges as an infrared quantity fixed by an entanglement-based equilibrium condition on the causal diamond of a late-time observer.

A central aspect of this resolution is the decoupling of vacuum energy from the infrared determination of Λ . Because the modular Hamiltonian is defined relative to a reference vacuum state, constant shifts of the stress-energy tensor cancel identically. This cancellation is exact and does not rely on symmetries, tuning, or modifications of Einstein's equations. In this precise sense, the usual radiative instability of the cosmological constant is absent.

The infrared equilibrium condition does not introduce new dynamical degrees of freedom or alter local gravitational dynamics. Einstein's equations remain valid at all scales. The additional input is a global infrared consistency requirement on physically admissible states, formulated in terms of relative entropy on a finite causal diamond. This requirement fixes the horizon scale in a

manner that is insensitive to ultraviolet physics but sensitive to the large-scale causal structure of spacetime.

An important feature of the construction is its limited dependence on detailed microscopic input. The existence and uniqueness of the infrared fixed point rely only on the universal area law for entanglement entropy and on the dilution of excitation energy in expanding cosmologies with a late-time event horizon. Precise numerical coefficients depend on the detailed definition of generalized entropy and on subleading corrections to the modular Hamiltonian, but these affect only order-unity factors and do not alter the qualitative result $\Lambda \sim H_0^2$.

The framework makes contact with a broader class of entanglement-based approaches to gravity, including entanglement equilibrium and thermodynamic derivations of Einstein's equations. The present work differs in that it does not aim to derive local field equations, but instead fixes a global infrared parameter that remains undetermined in standard semiclassical gravity.

Several open directions remain. A more explicit computation of the modular Hamiltonian for causal diamonds in general FLRW spacetimes would refine the order-unity coefficients appearing in the equilibrium condition. In addition, a systematic study of perturbations around the infrared fixed point may clarify potential observational signatures on horizon scales. These questions, however, do not affect the internal consistency or the central conclusion of the present analysis.

7 Conclusion

We have presented a self-contained resolution of the cosmological constant problem within semiclassical gravity based on an infrared entanglement equilibrium condition. The key result is that the cosmological constant is not a free parameter nor a vacuum energy contribution, but an emergent infrared scale fixed by the balance between modular excitation energy and generalized entropy on the causal diamond of a late-time observer.

The resulting cosmological constant is radiatively stable, insensitive to ultraviolet physics, and naturally of order the present Hubble scale, $\Lambda \sim H_0^2$, without fine-tuning or modification of Einstein's equations. The notorious hierarchy between the observed value of Λ and naive estimates of vacuum energy is shown to be physically irrelevant once the appropriate infrared notion of gravitational energy is employed.

More broadly, this work suggests that certain apparent naturalness problems in gravity may originate from an improper identification of the relevant degrees of freedom at large scales. When infrared entanglement and causal structure are properly taken into account, the cosmological constant ceases to be a problem requiring ultraviolet explanations.

Whether the infrared entanglement equilibrium principle admits independent observational tests remains an open question. In the absence of such deviations, the framework remains fully consistent with current observations while offering a conceptually economical resolution of one of the most persistent problems in theoretical physics.

The infrared entanglement equilibrium framework admits principled avenues for falsification. Any observational evidence for a cosmological constant that depends sensitively on ultraviolet vacuum contributions, or any confirmed departure from standard Λ CDM behavior on horizon scales incompatible with a finite causal-diamond infrared fixed point, would directly rule out the mechanism proposed here.

A Finite Causal-Diamond Averaging in Λ CDM

In this appendix we provide the technical justification for the statement that the excitation energy density averaged over a causal diamond in Λ CDM is naturally of order the present critical density.

A.1 Definition of the infrared density

Consider a spatially flat FLRW spacetime with metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2. \quad (21)$$

For a homogeneous matter density $\rho_m(t)$, we define the infrared excitation density as the spacetime average over the causal diamond \mathcal{D} ,

$$\rho_{\text{IR}} = \frac{1}{V_4} \int_{\mathcal{D}} d^4x \sqrt{-g} \rho_m(t), \quad V_4 = \int_{\mathcal{D}} d^4x \sqrt{-g}. \quad (22)$$

In FLRW coordinates this can be written as

$$\rho_{\text{IR}} = \frac{\int dt a^3(t) \rho_m(t) V_3(t)}{\int dt a^3(t) V_3(t)}, \quad (23)$$

where $V_3(t) = \frac{4\pi}{3} R^3(t)$ is the physical volume of the spatial slice of the diamond at time t .

A.2 Weighted average formulation

Changing variables to the scale factor using $dt = da/(aH(a))$, one finds

$$\rho_{\text{IR}} = \frac{\int da W(a) \rho_m(a)}{\int da W(a)}, \quad W(a) = \frac{a^2}{H(a)} R^3(a), \quad (24)$$

where $R(a)$ is the physical radius of the causal diamond and $H(a)$ the Hubble parameter.

The function $W(a)$ is purely geometric and encodes the spacetime volume weighting of each spatial slice.

A.3 Dominance of the maximal diamond slice

The radius $R(a)$ grows from zero at early times, reaches a maximum when the past- and future-directed null boundaries of the diamond intersect symmetrically, and then decreases again. Consequently, the weight function $W(a)$ is sharply peaked near the scale factor a_* corresponding to the maximal spatial slice of the diamond.

In Λ CDM with $\Omega_m + \Omega_\Lambda = 1$, this occurs near the matter- Λ transition, with

$$a_*^3 \sim \frac{\Omega_m}{\Omega_\Lambda}, \quad (25)$$

up to a numerical factor of order unity. As a result, the weighted average is dominated by values of $\rho_m(a)$ near a_* .

A.4 Order-of-magnitude estimate

Evaluating the matter density at a_* ,

$$\rho_m(a_*) = \rho_{m0} a_*^{-3}, \quad (26)$$

one finds

$$\rho_m(a_*) \sim \rho_{\text{crit},0}, \quad (27)$$

up to an order-unity coefficient depending on the precise values of Ω_m and Ω_Λ .

Corrections arising from the finite width of the peak in $W(a)$ modify this estimate only by numerical factors of order unity. No large or small hierarchies are generated. We therefore conclude that

$$\rho_{\text{IR}} = c \rho_{\text{crit},0}, \quad c = \mathcal{O}(1), \quad (28)$$

as stated in the main text.

B Modular Hamiltonian in FLRW Spacetimes

In this appendix we clarify the role of the modular Hamiltonian in the infrared entanglement equilibrium condition and explain why an exact expression for the modular Hamiltonian in general FLRW spacetimes is not required for the validity of the main results.

B.1 Exact results in flat spacetime

For a ball-shaped region of radius R in flat spacetime, the modular Hamiltonian associated with the vacuum of a conformal field theory is known exactly and takes a local form,

$$K_\sigma = 2\pi \int_{|\vec{x}| \leq R} d^3x \frac{R^2 - |\vec{x}|^2}{2R} T_{00}(\vec{x}). \quad (29)$$

For homogeneous excitations with energy density ρ , the corresponding modular energy scales as

$$\Delta \langle K_\sigma(R) \rangle = \frac{8\pi^2}{15} \rho R^4. \quad (30)$$

This result is universal and depends only on the local structure of the vacuum state near the boundary of the region.

B.2 Extension to curved backgrounds

In a curved spacetime, and in particular in an expanding FLRW universe, the exact modular Hamiltonian for a causal diamond is not known in closed form. However, several general properties remain robust:

- The modular Hamiltonian is defined *relative to the reference vacuum state*, ensuring exact cancellation of constant vacuum energy contributions.
- The leading contribution to the modular energy arises from short-distance correlations near the entangling surface, where curvature effects are parametrically suppressed.
- Curvature corrections enter as higher-derivative or nonlocal terms suppressed by powers of R^{-1} relative to the flat-space result.

As a consequence, for sufficiently large causal diamonds the modular energy retains the same infrared scaling as in flat spacetime,

$$\Delta\langle K_\sigma(R)\rangle = C_K \rho_{\text{IR}} R^4 \left[1 + \mathcal{O}\left(\frac{1}{R^2 H^2}\right) \right], \quad (31)$$

where C_K is a numerical coefficient of order unity and H is the Hubble parameter evaluated near the maximal slice of the diamond.

B.3 Dominance of the maximal spatial slice

The infrared entanglement equilibrium condition is controlled by the behavior of the modular energy near the maximal spatial slice of the causal diamond, where the physical radius R is largest and the geometry varies slowly on the scale R . In this regime, the local structure of the modular Hamiltonian is well approximated by the flat-space expression, with curvature effects producing only order-unity corrections to numerical coefficients.

Crucially, the equilibrium condition depends on the *scaling* of the modular energy with R , not on its precise normalization. As long as $\Delta\langle K_\sigma(R)\rangle \propto \rho_{\text{IR}} R^4$ at leading order, the existence, uniqueness, and infrared location of the equilibrium point R_* are unaffected.

B.4 Implications for the infrared equilibrium

The absence of an exact modular Hamiltonian for general FLRW spacetimes therefore does not represent an obstruction to the present framework. All ultraviolet and curvature-sensitive contributions either cancel identically in the vacuum subtraction or enter only as subleading corrections that modify order-unity coefficients.

A full computation of the exact modular Hamiltonian in FLRW spacetimes would be of intrinsic interest and could refine numerical prefactors in the equilibrium condition. However, it is not required for the infrared determination of the cosmological constant presented in this work.

This completes the justification of the approximations employed in the main text.